LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc., DEGREE EXAMINATION – **STATISTICS**

THIRD SEMESTER – **NOVEMBER 2013**

ST 3503/ST 3501/ST 3500 – STATISTICAL MATHEMATICS-II

Date : 08/11/2013 Dept. No. Max. : 100 Marks
Time : 9:00 - 12:00

PART – A

Answer **ALL** the questions (10 x 2 = 20 marks)

1. Define Riemann integrable function.
2. For the p.d.f. $f(x)=\frac{c}{x^3}$, $x \geq 1$, find the value of c.
3. Verify that the integral $\int_1^\infty \frac{dx}{x^2}$ converges.
4. Define Gamma function.
5. Evaluate $\int_0^a \int_0^b (x^2 + y^2)\, dx\, dy$.
6. Express the jacobian of u, v with respect to x, y.
7. Solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 4y = 0$.
8. Define Laplace transform of a function $f(t)$.
9. Write the system of linear Homogeneous equation.
10. State Cayley – Hamilton Theorem.

PART – B

Answer any **FIVE** questions (5 x 8 = 40 marks)

11. Discuss the convergence of

    i) $\int_0^\infty e^{-x} dx$ ii) $\int_0^1 \frac{1}{x}\, dx$.

12. Show that a continuous function on [a,b] is Riemann – inteqrable on [a,b].
13. Derive the M.G.F. of $f(x, \theta) \begin{cases} \theta . e^{-\theta x}, x \geq 0 \\ 0 \quad , otherwise \end{cases}$

    and hence find Mean and variance.
14. Discuss the convergence of Gamma integral.
15. Evaluate $\iint (x^2+y^2)\, dx\, dy$ over the region for which x, y are each ≥ 0 and $x+y \leq 1$.
16. Soule $(3D^2+D-14)\, y=13\, e^{4x}$.
17. If L {f(t)} = F(s), then prove that L {t f(t)} = $-\frac{d}{ds}$F(s). Deduce the result for L { t^n.f(t)}.
18. Determine the characteristic roots of the Matrix $A=\begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & -1 \\ 2 & -1 & 0 \end{bmatrix}$.

PART – C

Answer any **TWO** questions: (2 x 20 = 40 marks)

19. a) if $f \in R[a,b]$ and $g \in R[a,b]$, show that $f+g \in R[a,b]$ and that $\int_a^b (f+g) = \int_a^b f + \int_a^b g$.

b) Derive the mean and variance of uniform distribution over [a b].

20. a) Prove that $\beta(m,n) = \frac{\Gamma(m)\cdot\Gamma(n)}{\Gamma(m+n)}$.

b) Two random variables X, Y have the joint p.d.f $f(x,y) = \begin{cases} 2-x-y, & 0 \le x \le 1,\ 0 \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$

Find the covariance between X and Y.

21. a) Solve $(D^2 - 13D + 12)\,y = e^{-2x} + 5e^{x}$

b) Solve the equation $\frac{d^2y}{dt^2} + t\frac{dy}{dt} - y = 0$ if $y(0)=0$ and $y'(0) = 1$.

22. a) Find the inverse of the following Matrix by using Cayley – Hamilton theorem:

$$A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}.$$

b) Solve completely the system of equations:

$$\begin{aligned} x+3y-2z &= 0 \\ 2x-y+4z &= 0 \\ x-11y+14z &= 0 \end{aligned}.$$

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